Computer Simulation and Applications
Assignment No 3

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September 17, 2008
Soft deadline: September 23 at 11 hrs.
Hard deadline: September 24, 24.00 hrs.

Number of credit points is 5. Refer any questions you might have to deutz@liacs.nl.

1 Assignments

1. Describe at least one question you posed yourself in your study of the logistic family $f(x) = ax(1-x)$ and describe how you went about to get the answer.

2. Consider the map $f(x) = x^3 + x$. Domain=Range=$\mathbb{R}$. Determine the fixed points of this map. Decide whether they are sinks or sources. You will have to work without the Theorem of the 9-September lecture stated below, since it does not apply.

THM 1 Let $f$ be a differentiable map on $\mathbb{R}$, and assume that $p$ is a fixed point of $f$. Then:

a) If $|f'(p)| < 1$, then $p$ is a sink.

b) If $|f'(p)| > 1$, then $p$ is a source.

3. Is the period-two orbit of the map $f(x) = 2x^2 - 5x$ on $\mathbb{R}$ a sink, a source, or neither. Use the Theorem of the 16-September lecture for the classification.

4. Consider the map $f(x) = 4x(1-x)$. Domain=Range=$[0,1]$.

a) Find the period-2 orbit and determine whether it is a sink, source or neither. Use the theorem of the lecture.

b) Prove that for each positive integer $k$, there is an orbit of period $k$.
   In the Fig. 4 the graph of the map $f$ and its second and third iterate are drawn.

c) for each $k, 0 \leq k \leq 10$ compute the number of fixed points of $f^k$, the number of fixed points of $f^k$ due to lower period orbits, and the number of orbits of period $k$. Make a table.
Write a computer program with the goal of redoing the tables we did for $3.3x(1 - x)$ and $3.5x(1 - x)$. What periodic behavior emerges? Try several initial conditions to explore the basin of the attracting periodic behavior. In addition let your program choose the different initial values with a (pseudo-)random number generator. Then try different values for $a < 3.57$. Report your results.

Consider the bifurcation diagram (see Fig. 6) we discussed in class. The parameter $a$ is bigger than 0 and less than 4. For each member of the logistic family $ax(1 - x)$, domain=range=[0,1]. Write a program which will generate this diagram. As we discussed in class the diagram can be generated as follows: for each parameter value $a$, choose a (pseudo-)random number as the initial value. Iterate this random initial value for a large number of times. Delete say the first one hundred iterates. The remaining iterates are plotted. Increment the parameter value $a$ by a fixed, chosen step size. For the incremented parameter value you repeat the above: a random initial value is chosen, a large number of iterates is computed, the first one hundred iterates are discarded, the remaining are plotted. Etc. etc.

Assignments suggested by the lecture of September 16

1. In the lecture we discussed and proved the following theorem. Let $P(x)$ be a polynomial (examples of polynomials are: $-x^4 + \sqrt{2}x^2 - 5x + 3$, or $x^{1000} + 3.1x^3 - 10$. If $r$ is a root of $P$ (that is, $P(r) = 0$, then $P(x) = (x - r) \cdot Q(x)$ for some polynomial $Q(x)$. The degree of $Q(x)$ will be
necessarily lower than the degree of $P(x)$. The latter comes in handy in solving polynomial equations.

a Write up the proof of this theorem.

b Show how it comes in handy in the business of solving polynomial equations.